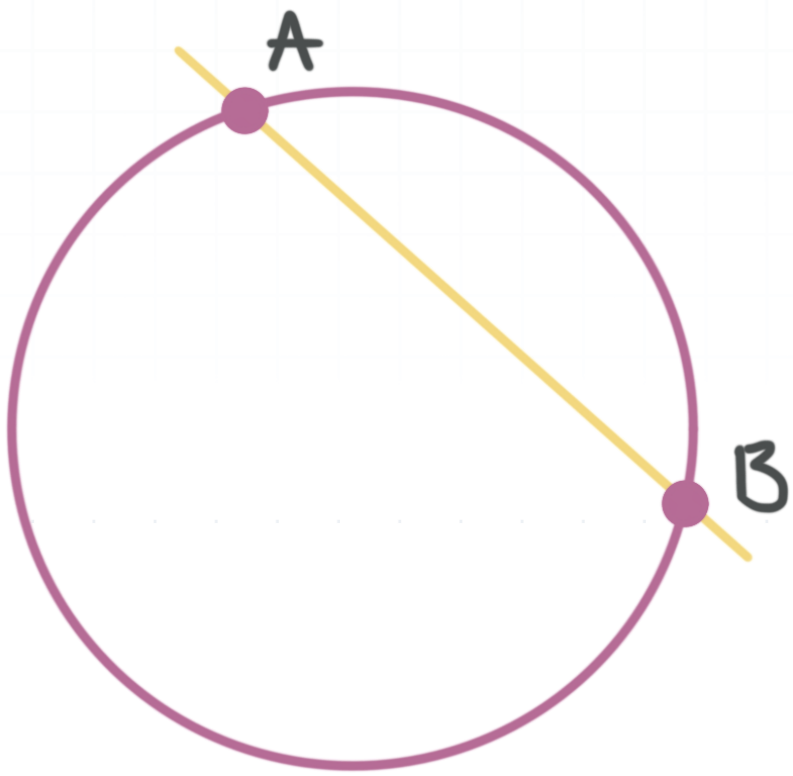
Intersecting tangents and secants

In this lesson we'll look at the relationships formed from intersecting tangents and secants in circles.

Secant

The **secant of a circle** is a line or line segment that intersects the circle at two points. $\overline{AB}$ is a secant of this circle.



Intersecting secants theorem

There's a special relationship between two secants that intersect outside of a circle. The length outside the circle, multiplied by the length of the whole secant is equal to the outside length of the other secant multiplied

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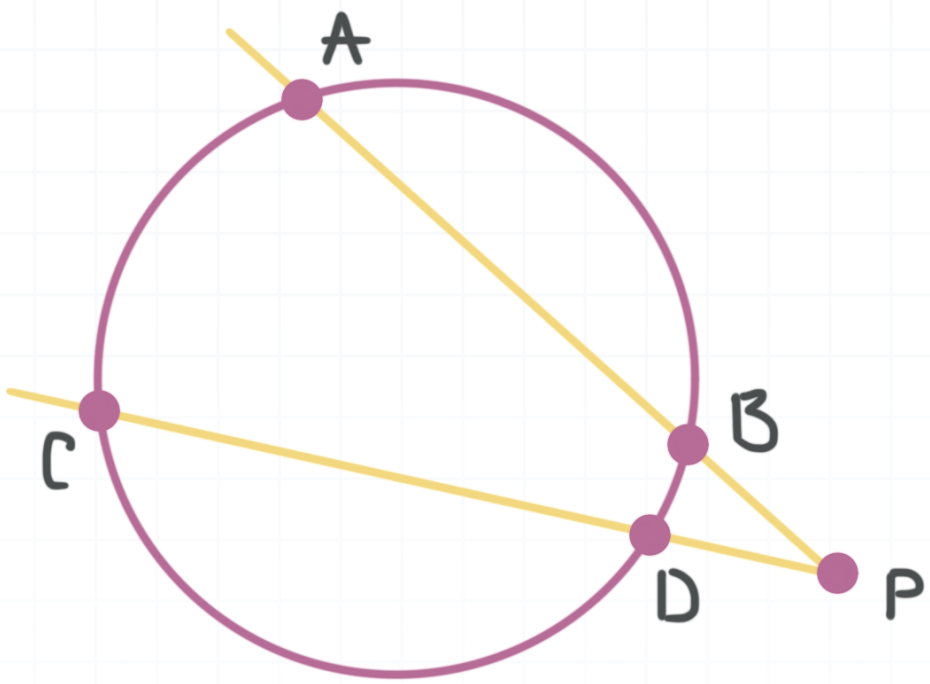
by the whole length of the other secant. In the following circle, the secants $\overline{AP}$ and $\overline{CP}$ intersect at point P,



so

$$BP \cdot AP = DP \cdot CP$$

Tangents

The **tangent of a circle** is a line or line segment that intersects a circle at only one point. $\overleftrightarrow{AB}$ is a tangent of this circle.

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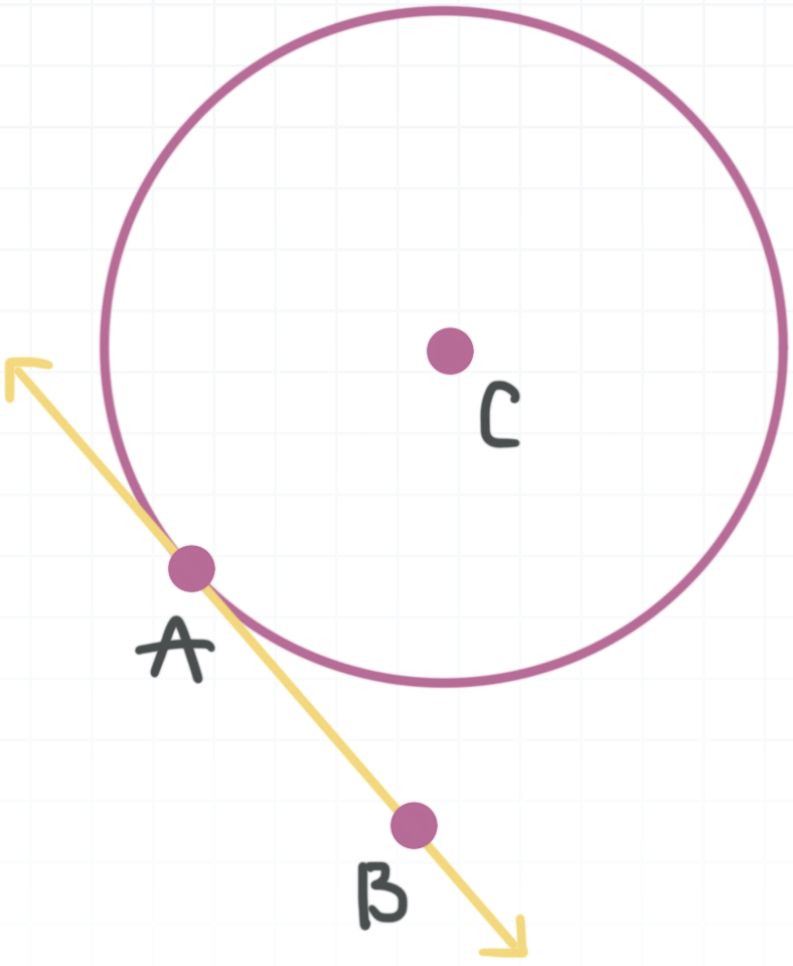

Intersecting tangent-secant theorem

There is also a special relationship between a tangent and a secant that intersect outside of a circle. The length of the outside portion of the tangent, multiplied by the length of the whole secant, is equal to the squared length of the tangent. Tangent $\overleftrightarrow{AP}$ and secant $\overline{DP}$ intersect at point P,

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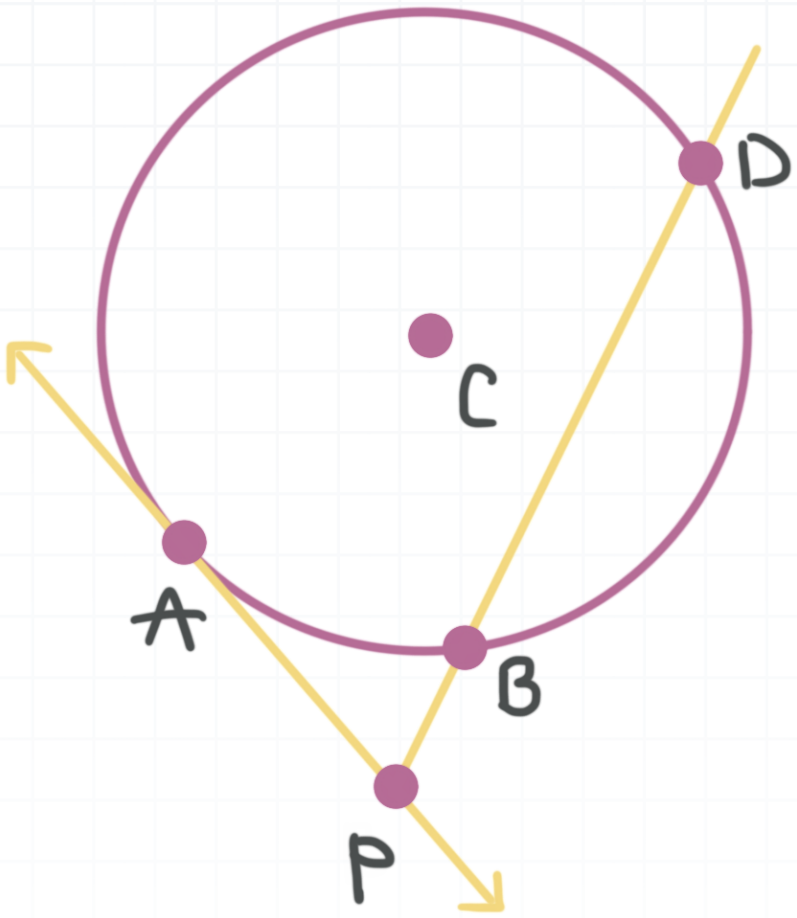

so

$$AP^2 = DP \cdot BP$$

Let's start by working through an example.

Example

Solve for the value of x in the figure, assuming DP is a tangent.

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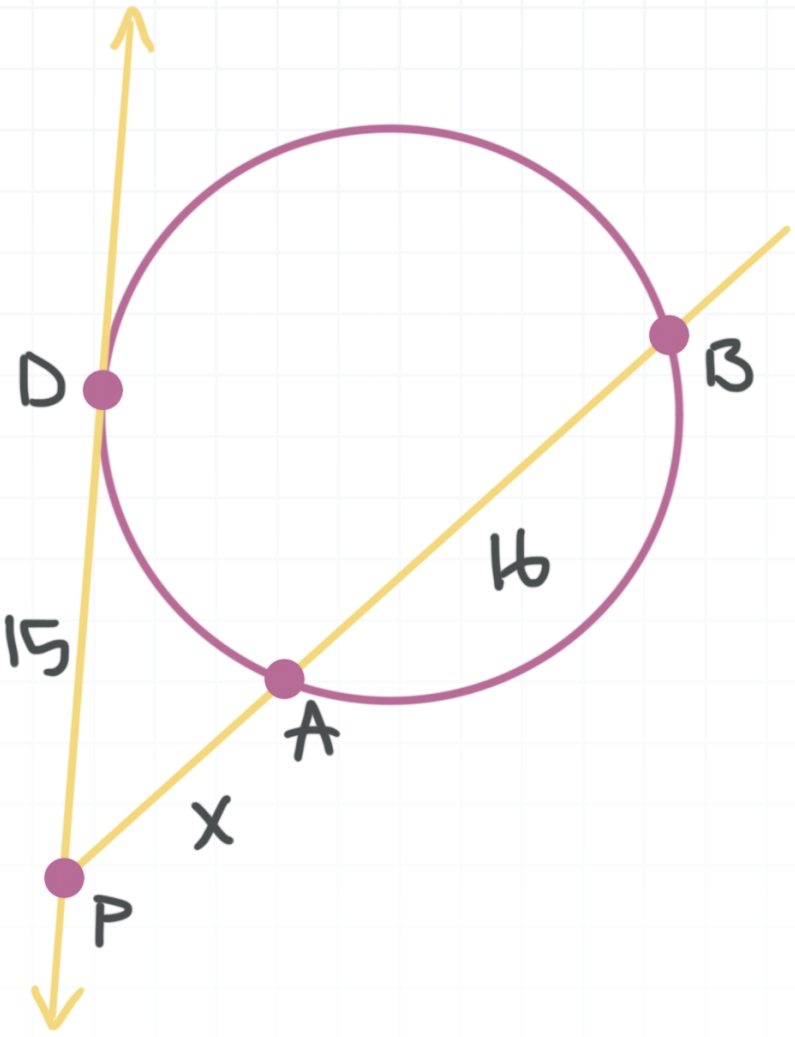

Because there's a secant intersecting with a tangent, we can follow the formula and plug in the values from the figure.

$$\text{outside} \cdot \text{whole} = \text{tangent}^2$$

$$x(x+16) = 15^2$$

$$x^2 + 16x = 225$$

$$x^2 + 16x - 225 = 0$$

$$(x+25)(x-9) = 0$$

$$x = -25 \text{ or } x = 9$$

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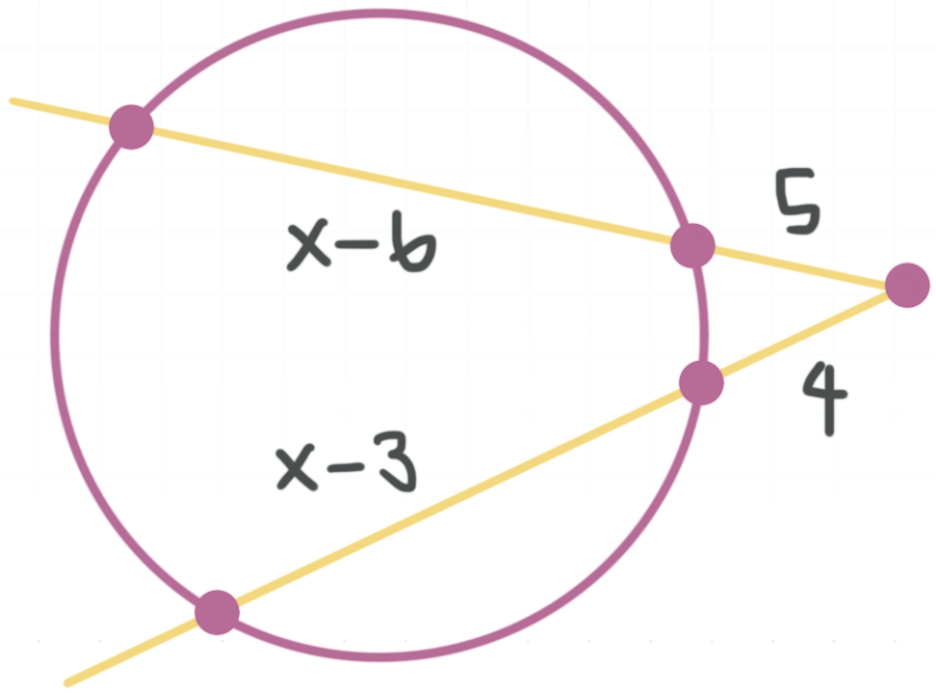
A line segment can't have a negative length, so rule out $x = -25$ and conclude that $x = 9$.

Let's do one more problem.

Example

Given the lengths in the figure, solve for x.



Because there are two secants intersecting, we can follow the formula and plug in the values from the figure.

$$\text{outside} \cdot \text{whole} = \text{outside} \cdot \text{whole}$$

$$5(x - 6 + 5) = 4(x - 3 + 4)$$

$$5(x - 1) = 4(x + 1)$$

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$$5x - 5 = 4x + 4$$

$$x - 5 = 4$$

$$x = 9$$

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